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DEPARTMENTS.

DISCUSSION.

THE EVALUATION OF $\int_0^\pi \frac{\sin mx}{x} dx$.

By S. A. COREY, Hiteman, Iowa.

The following solution of problem 203, Calculus, the evaluation of the definite integral $\int_0^\pi \frac{\sin mx}{x} dx$ (m an integer), involves several points of interest.

$$\int_0^\pi \frac{\sin mx}{x} dx = \int_0^{m\pi} \frac{\sin x}{x} dx.$$

Developing $\int \frac{\sin x}{x} dx$ by the formula,*

$$\begin{aligned} f(x) = & f(0) + \frac{x}{r^2} \left\{ [f'(x) + f'(0)] + 2 \left[f' \left[\frac{x}{r} \right] + f' \left[\frac{2x}{r} \right] + f' \left[\frac{3x}{r} \right] + \dots \right. \right. \\ & \left. \left. + f' \left[\frac{r-1}{r} x \right] \right] \right\} - \frac{B_1 x^2}{r^2 \cdot 2!} [f''(x) - f''(0)] + \frac{B_2 x^4}{r^4 \cdot 4!} [f^{(4)}(x) - f^{(4)}(0)] \\ & - \frac{B_3 x^6}{r^6 \cdot 6!} [f^{(6)}(x) - f^{(6)}(0)] + \dots + (-1)^n \frac{B_n x^{(2n)}}{r^{(2n)} \cdot (2n)!} [f^{(2n)}(x) - f^{(2n)}(0)] + \dots \quad (1), \end{aligned}$$

(B_1, B_2, B_3, \dots , being Bernoulli's numbers), and taking $r=2m$, we get

$$\begin{aligned} \int \frac{\sin x}{x} dx = & c + \frac{x}{(2m) \cdot 2!} \left\{ \left[\frac{\sin x}{x} + 1 \right] + 2 \left\{ \frac{\sin \left[\frac{x}{2m} \right]}{\frac{x}{2m}} + \frac{\sin \left[\frac{2x}{2m} \right]}{\frac{2x}{2m}} + \frac{\sin \left[\frac{3x}{2m} \right]}{\frac{3x}{2m}} \right. \right. \\ & \left. \left. + \dots + \frac{\sin \left[\frac{2m-1}{2m} x \right]}{\frac{(2m-1)x}{2m}} \right\} \right\} - \frac{x^2}{6 \cdot (2m)^2 \cdot 2!} \left[\frac{x \cos x - \sin x}{x^2} \right] \\ & + \frac{x^2}{30 \cdot (2m)^4 \cdot 4!} \left[\left(\frac{6-x^2}{x^3} \right) \cos x + \left(\frac{6-3x^2}{x^4} \right) \sin x \right] \dots \quad (2). \end{aligned}$$

But as m is an integer, $\int_0^{m\pi} \frac{\sin x}{x} dx$ develops by means of (2) into

$$\frac{1}{4}\pi + \left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots - (-1)^m \frac{1}{(2m-1)}\right) \pm \pi \left(\frac{1}{6 \cdot 2^2 \cdot m \cdot 2!} + \frac{(m\pi)^2 - 6}{30 \cdot 2^4 \cdot m^3 \cdot 4!} + \frac{(m\pi)^4 - 20(m\pi)^2 + 120}{42 \cdot 2^6 \cdot m^5 \cdot 6!} + \dots\right) \dots (3),$$

according as m is odd or even.

For convenient use in numerical computation (3) may be put into the form

$$\int_0^{m\pi} \frac{\sin x}{x} dx = \frac{1}{4}\pi + \left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots - (-1)^m \frac{1}{(2m-1)}\right) \pm \left(\frac{c_1}{m} - \frac{c_3}{m^3} + \frac{c_5}{m^5} - \dots\right) \dots (4),$$

where $c_1 = .0682995$, $c_3 = .0019567$, $c_5 = .0001948$, approximately.

By means of (4) the values of the definite integral corresponding to a few values of m are readily found to be as follows:

$$\begin{aligned} \text{For } m=1, & 1.851936 - \\ m=2, & 1.418158 + \\ m=3, & 1.674760 - \\ m=4, & 1.492164 + \\ m=5, & 1.633963 - \\ m=6, & 1.518036 - \\ & \dots \end{aligned}$$

SOLUTIONS OF PROBLEMS.

ALGEBRA.

247. Proposed by PROFESSOR G. W. GREENWOOD, M. A., McKendree College, Lebanon, Ill.

Find the sum, to n terms, of

$$1 + \frac{n}{2} + \frac{n(n+2)}{2 \cdot 4} + \frac{n(n+2)(n+4)}{2 \cdot 4 \cdot 6} + \dots$$

I. Solution by the PROPOSER.

The series is the coefficient of x^{n-1} in $(1-x)^{-\frac{1}{2}n}(1-x)^{-1}$; i. e., in $(1-x)^{-(\frac{1}{2}n+1)}$. Hence the required sum is

$$\frac{(n+2)(n+4) \dots (3n-2)}{2 \cdot 4 \dots (2n-2)}.$$